Quantum Mechanics A
Fall 2010
Instructor: Professor E. Manousakis

Mid-Term Exam

Choose only two out of the three problems below, circle the chosen problems and provide your solution. All problems are worth the same.

Problem 1

A particle of mass m is under the influence of a 1D potential of the form (see also Fig. 1)

$$V(x) = \left\{ \begin{array}{ll} -V & 0 \leq |x| < a \\ 0, & |x| \geq a \end{array} \right\} \tag{1}$$

where $V > 0$.

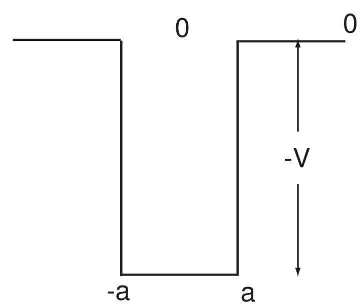

Figure 1:

a) Find the equations that determine the negative energy eigenvalues which correspond to even and odd energy eigenfunctions. (5 pts)

b) Is the ground state an even or an odd solution? (1 pt)

c) In the large V limit, i.e., when $\epsilon = \sqrt{\hbar^2/(2mVa^2)} << 1$ show that the ground state energy is given by

$$E = \frac{\hbar^2 p_0^2}{2m} - V, \tag{2}$$

where p_0 to first order in ϵ can be approximated by

$$p_0 \simeq \frac{\pi}{2a}(1-\epsilon). \tag{3}$$

(4 pts)

Problem 2

A particle of mass m is under the influence of a 1D potential shown in Fig. 2, i.e., of the form

$$V(x) = \left\{ \begin{array}{cc} V\delta(x) & 0 \leq |x| < a \\ \infty, & |x| \geq a \end{array} \right\} \tag{4}$$

where $V > 0$. More simply the particle is in an infinite square well with a delta function at the center of the well.

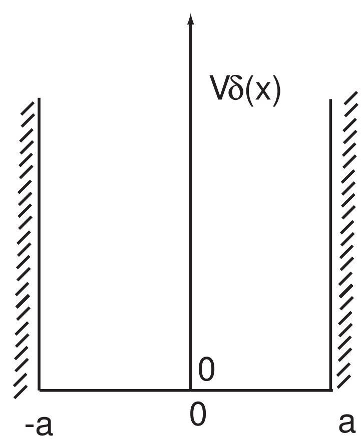

Figure 2:

a) Find the equation that determines the energy eigenvalues which correspond to even energy eigenfunctions. (6 pts)

b) In the large V limit, i.e., when $\epsilon = \hbar^2/(mVa) << 1$ show that the ground state energy is given by

$$E = \frac{\hbar^2 k_0^2}{2m}, \tag{5}$$

where k_0 to first order in ϵ can be approximated by

$$k_0 \simeq \frac{\pi}{a}(1-\epsilon). \tag{6}$$

(4 pts)

Problem 3

A two-state system (such as the problem of photon-polarization, or the last problem in the last homework assignment) is described by a Hamiltonian matrix

$$\hat{H} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}, \tag{7}$$

where $\alpha = \langle 1|\hat{H}|1\rangle = \langle 2|\hat{H}|2\rangle$ are the diagonal matrix elements of the Hamiltonian in the basis defined by states $|1\rangle$ and $|2\rangle$, and β is a real number and it gives the off-diagonal matrix elements of the Hamiltonian in this basis, i.e., $\beta = \langle 1|\hat{H}|2\rangle = \langle 2|\hat{H}|1\rangle$.

a) Find the eigenstates and eigenvalues of this Hamiltonian as a linear combination of the states $|1\rangle$ and $|2\rangle$. (3 pts)

b) Assume that at time $t = 0$ the system is in state $|1\rangle$, i.e.,

$$|\psi(t=0)\rangle = |1\rangle. \tag{8}$$

Find the state of the system at a later time t. (4 pts)

c) What is the probability to find the system in state $|2\rangle$ if observed at time t, given that it was in state $|1\rangle$ at time $t = 0$. (3 pts)